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Vacuum selection by recollapsing

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Abstract

We discuss the possibility that the vacuum is dynamically determined in the history of the universe. The point is that some of the bubbles with a certain vacuum shrink by the evolution of the universe via gravity and may become black holes. When the temperature of the phase transition T_{PT} is higher than 10^9 GeV, these black holes evaporate until now. If $T_{PT} < 10^9$ GeV, we may see these black holes in our universe. It is interesting that in many cases false vacua are favored in the context of cosmology. By using this argument, supersymmetry(SUSY), if it exists, can be broken cosmologically. We can guess the SUSY breaking scale from the mass of the black holes.

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1 Introduction

It is well known that the cosmology is strongly related with the particle physics. [1] The cosmology gives various constraints to the particle physics. For examples, the present ratio of the radiation density and the critical density gives constraints to the mass and the interaction of the stable particles and the success of the Big-Bang Nucleosynthesis requires shorter lifetime than about one second for the unstable particles with large number density. On the other hand, elementary particle physics also gives various effects to the cosmology. In order to examine the early universe, we need the knowledge of the elementary particle physics, because the evolution of the universe is strongly dependent on the physics at the small scale. More concretely the elementary particle theory gives some candidates for the dark matter and the concept of the field theory is applied to solve some cosmological problems as in inflation. [2]

Are there any possibility that the present elementary particle theory is more dynamically determined by the history of the universe? This is the point which we examine in this paper.

Before discussing cosmological situation, let us roughly recall the theory of evolution for creatures. Every creatures have genes. The genes essentially determine the behavior of the creatures. The circumstance around the creatures select the creatures by their behaviors. If some creatures behave more suitably for their circumstance, then the creatures have longer life and create more children and their genes. Namely genes with longer life survive in the history.

What we would like to do is to apply the above logic to the elementary particle physics. The elementary particle theories determine the evolution of the universes. Some of the universes are expanding and some of them are recollapsing. As the result, the universes with longer life survive. In other words, theories which make the universe have longer life survive. What are the features of theories with long or short life? This is main motivation in this paper.

But what does 'many' universes mean? The answer in this paper is that it means many bubbles with different vacua which appear at the phase transition. In the early universe, there must be several phase transitions. If there are several minima of the potential at the phase transition, there may be several kinds of bubbles with different vacua in the universe simultaneously. Since generally the bubbles with different field theories evolve differently, the bubbles with different vacua may evolve differently. Some of them may expand more rapidly than the other bubbles and dominate the other universes, and some of them may shrink and become black holes when the radius of the bubbles becomes smaller than the Schwarzschild radius. As the result, we have many universes with different lifetimes and the selection of the vacua happens.

The vacuum selection by using inflation has been argued in the literature. [2, 3, 4] A. Linde argued that the initial condition of the vacuum expectation value of the scalar is naturally selected so that the inflation happens, because

inflated universe dominates the universe. After his argument, various features have been discussed in the context of the inflation. However, in a sense this selection is not true selection, because the universes without inflation also survive unless the universes are recollapsing, though they occupy only smaller region in the whole universe. Therefore it is possible that we are living in one of these universes, though the simple argument of probability teaches us that we need the strong luckiness or the anthropic principle.

On the other hand, as long as we know, there are only a few papers on the vacuum selection by using recollapsing via gravity force. [5] Banks et al. have argued this possibility in the context of the string moduli problem. They argued that the universes with negative cosmological constant shrink following Friedmann equation. By this argument, they insisted that the universes with negative cosmological constant are unstable.

We think that it is important to examine more generally the features of universes which shrink by gravity effect. By studying on the universes with short lifetime, we can understand the features of universes with long life and the features of our particle field theories which we have in our universe. In this paper, under simple assumptions we examine several cases in which the vacuum selection by recollapsing happens. It is interesting that in many cases false vacua are selected by this argument. Namely, cosmology prefers local minima. And we insist that the recollapsing bubbles become black holes. If the black holes evaporates until now, this selection by recollapsing becomes true selection. In other words, the bubble with the special vacuum is too weak to survive. It is also interesting that we may be able to observe the other universes as the black holes in our horizon unless the black holes evaporate until now. Moreover, we discuss the application of the cosmological vacuum selection to the elementary particle theories, especially to the cosmological supersymmetry(SUSY) breaking.

The plan of this paper is as follows. After the review on the evolution of the universe in section 2, we examine several cases in which the selection by recollapsing happens in section 3. In section 4, we discuss various features on the primordial black holes which are formed by the recollapsing via gravity force. In section 5, we apply the result that cosmology prefers the local minima to the particle theory. Especially we examine the possibility that the SUSY is cosmologically broken. Finally we discuss our assumptions and summarize our arguments.

2 Evolution of the Universe

If the universe is homogeneous and isotropic, the metric can be written in the form

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \quad (2.1)$$

which is Robertson-Walker metric. Under this metric, the Einstein equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu} + \Lambda g_{\mu\nu}, \quad (2.2)$$

where $G_{\mu\nu}$, $R_{\mu\nu}$, $T_{\mu\nu}$ and Λ are the Einstein tensor, the Ricci tensor, the stress-energy tensor for all the fields and a cosmological constant, respectively, leads to the Friedmann equation

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = -\frac{k}{a^2} + \frac{8\pi G}{3}\rho \quad (2.3)$$

and the energy equation

$$d(\rho a^3) = -pd(a^3), \quad (2.4)$$

where H is the Hubble constant. Here we take

$$\rho = \bar{\rho} + \rho_\Lambda = \rho_R + \rho_M + \rho_\Lambda \quad (2.5)$$

$$p = \bar{p} + p_\Lambda = p_R + p_M + p_\Lambda \quad (2.6)$$

$$T_\nu^\mu = \text{diag}(\bar{\rho}, -\bar{p}, -\bar{p}, -\bar{p}) \quad (2.7)$$

$$\rho_\Lambda = -p_\Lambda \equiv \frac{\Lambda}{8\pi G}, \quad (2.8)$$

where $\rho_{(R,M)}$ and $p_{(R,M)}$ are the energy density and pressure for (relativistic fields(radiation), non-relativistic fields(matter)), respectively. Though the energy density ρ and the pressure p are the sum of the contribution from the relativistic fields, non-relativistic fields and the cosmological constant, the energy equation is realized independently if the energy transfer among relativistic fields, non-relativistic fields and the cosmological constant is negligible. Namely

$$d(\rho_A a^3) = -p_A d(a^3), \quad (2.9)$$

where $A = R, M, \Lambda$. When

$$p_A = \gamma_A \rho_A, \quad (2.10)$$

the differential equation (2.9) can be solved as

$$\rho \propto a^{-3(1+\gamma_A)}. \quad (2.11)$$

Here $\gamma_R = 1/3$, $\gamma_M = 0$ and $\gamma_\Lambda = -1$. Then the Friedmann equation is rewritten as

$$-\frac{1}{2}k = \frac{1}{2}\dot{a}^2 + V(a) \quad (2.12)$$

$$V(a) = -\frac{4\pi G}{3}a^2\rho(a), \quad (2.13)$$

which has the same kinematics as of one dimensional particle moving with the energy $E = -k/2$ under the potential $V(a)$. In the following analysis, we take the initial condition $\dot{a} > 0$, namely the universe is expanding in the beginning.

When $\rho_\Lambda = 0$, the global behavior is determined by the signature of the parameter k . If k is positive, the universe becomes recollapsing. If k is zero or negative, the universe keeps expanding.

On the other hand, if the vacuum energy dominates the energy density, the signature of the cosmological constant plays essential roles. If the cosmological constant is positive, the universe is expanding exponentially. This situation is called inflation. If the cosmological constant is negative, the universe becomes recollapsing.

Let us estimate the time scale for recollapsing by solving the Friedmann equation (2.12). With vanishing cosmological constant, the time scale $\tau_{(R,M)}$ for the (Radiation, Matter) dominated case is calculated as

$$\tau_R = \frac{1}{H_I} \left(\frac{2}{\Omega_I - 1} + O(1) \right) = \frac{O(t_I)}{(\Omega_I - 1)} \quad (2.14)$$

$$\tau_M = \frac{1}{H_I} \left(\frac{\pi}{(\Omega_I - 1)^{3/2}} + O((\Omega_I - 1)^{-1}) \right) = \frac{O(t_I)}{(\Omega_I - 1)^{3/2}} \quad (2.15)$$

where $\Omega \equiv \rho/\rho_c$ is defined by using critical density $\rho_c \equiv 3H^2/(8\pi G)$ and the suffix I represents the initial value at the time t_I . We can easily find that when $\Omega - 1 = O(1)$, the time scale for recollapsing is of order $t_I \sim 1/H_I$. This estimation show that the universe recollapses soon after $\Omega - 1$ becomes of order 1.

With the negative cosmological constant, the time scale τ_{ADS} for recollapsing is

$$\tau_{ADS} \sim 2 \int_{a_I}^{a_{max}} da \frac{1}{\sqrt{-k - 2V(a)}} = \frac{1}{H_I \sqrt{-\Omega_I}} \left(\pi - 2 \text{Arcsin} \left(\sqrt{\frac{-\Omega_I}{1 - \Omega_I}} \right) \right) = O(t_I), \quad (2.16)$$

where a_{max} is determined by the relation $-k = 2V(a_{max})$. This means that the universe recollapses soon after the vacuum energy dominates the energy density of the universe. The results in this section are summarized in table 1.

| | $k > 0 (\Omega > 1)$ | $k = 0 (\Omega = 1)$ | $k < 0 (\Omega < 1)$ |
|-------------------------------|-----------------------------|----------------------|----------------------|
| $\Lambda > 0$ | - | inflation | inflation |
| $\Lambda = 0 (\rho = \rho_R)$ | $O(t_I)(\Omega - 1)^{-3/2}$ | ∞ | ∞ |
| $\Lambda = 0 (\rho = \rho_M)$ | $O(t_I)(\Omega - 1)^{-1}$ | ∞ | ∞ |
| $\Lambda < 0$ | $O(t_I)$ | $(O(t_I))$ | $(O(t_I))$ |

table 1. The lifetime for recollapsing

3 Examples

In this section we discuss several examples in which collapsing bubbles and expanding bubbles exist simultaneously, namely vacuum selection by recollapsing occurs.

It is not so easy to understand the evolution of the universe at the phase transition, because generally the universe is not homogeneous nor isotropic. It is also difficult to solve the Einstein equation in such general cases, therefore we take several assumptions.

The first assumption is that the Friedmann equation can be used for estimating the evolution of the bubble universes. If the size of the bubbles is larger than the horizon length $d_H \sim 1/H$ and the space inside the bubble is almost homogeneous and isotropic, then it is natural to use the Friedmann equation for the evolution of the space inside the bubble. Such calculation is adopted by several people who discussed the evolution of the universe, [2, 3, 4, 5] for example, in the context of the chaotic inflation by A. Linde. Here we use the Friedmann equation for the evolution of the bubble universes.

The second assumption is that the phase transition does not change the energy density. The phase transition can change only the carrier of the energy among relativistic particles, non-relativistic particles and vacuum. This is only for the simplicity.

We consider the following situation. At the higher temperature, there is only one minimum, which disappears and two new minima appear around the critical temperature T_{PT} . Suppose that the two phases coexist for a while after the phase transition and the time scale of the coexistence is larger than the typical time scale for recollapsing, e.g., $O(t_I)$. Under these assumptions, we examine several cases in which collapsing bubbles and expanding bubbles exist at the same time.

The first case is that the universe has the larger energy density than the critical density ($k > 0$), therefore $\Omega_{PT} > 1$ at the time t_{PT} when the phase transition occurs. After the phase transition, the potential has one global minimum with vanishing cosmological constant and the other minimum which is local. The bubble universe with the true vacuum recollapses because the energy density is larger than the critical density. The time scale for the recollapsing is $O(t_I)$ unless the energy density is tuned to the critical density. It is natural that the recollapsing bubbles become black holes, which we will discuss later. On the other hand, the bubble universe with the false vacuum inflates if the cosmological constant dominates the energy density before reaching the time when the bubble begins to shrink.

In order to regard the inflating universe as our universe, the inflation must stop some time because our universe does not inflate at present, namely cosmological constant must vanish till now. Here we expect that there is an unknown mechanism which realizes the vanishing cosmological constant. And such a mechanism automatically stop the inflation and the vacuum energy is released in the

universe(reheating). Of course we have no reliable mechanism for the vanishing cosmological constant, so we do not discuss more on how to stop the inflation here.

The second case is that the energy density is smaller than the critical density. After the phase transition, the local minimum has vanishing cosmological constant, and the global minimum has negative cosmological constant. After the negative cosmological constant in the bubble universe of the true vacuum dominates the energy density of the universe, the bubble universes begin to shrink. Therefore the bubble universes with true vacuum become black holes and the bubble universe with the false vacuum keeps expanding and dominates our universe.

In the above two cases, our universe is on the false vacuum. The false vacuum decays to the true vacuum by quantum tunneling effect. [6, 7] The lifetime of the universe must be longer than the age of the present universe if you would like to regard the universe as our universe. We will shortly discuss the lifetime of the universe.

Of course, we can think about the intermediate situations between the above two cases. Namely, the potential has a global minimum with the negative cosmological constant and a local minimum with the positive cosmological constant. In this case, the bubble universes with the true vacuum become black holes and the bubble universes with the false vacuum inflate. Notice that the above vacuum selection happens even if the initial number density of the bubble universes with the false vacuum is much smaller than that of the true vacuum. Since the bubble universes with true vacuum are recollapsing, the tiny part of the universe with the false vacuum can dominate the whole universe if it exists.

The final case is that the energy density is almost the same as the critical density, but a little bit larger than the critical density. And the both vacua are degenerate with the vanishing cosmological constant. Therefore, bubbles with both vacua turn to recollapse some time. Moreover, we take the situation that one of the vacua gives the matter dominated universe, and the other gives the radiation dominated universe. Here for simplicity, soon after the phase transition, such a difference is realized. Then the time scales for recollapsing are different and dependent on whether the bubble are the radiation dominant or matter dominant. As we discussed the lifetime in the previous section, the matter dominated universe has longer lifetime than the radiation dominated universe under the same initial conditions. If the energy density is near the critical density, the difference becomes large, because the lifetime is proportional to $1/(\Omega - 1)$ for the radiation dominated universe and $1/(\Omega - 1)^{\frac{3}{2}}$ for the matter dominated universe. If the radiation dominated universe become black hole with shorter period than the age of our universe and the matter dominated universe has longer lifetime than the age of our universe, then the vacuum selection is realized. Since $\Omega \sim 1$ is realized after inflation, it may be possible that even in our real world, the

situation happens.

By examining the above cases and the other cases of vacuum selection [2, 4] by inflation, we can easily conclude that the cosmology prefers local minima. However, in the ordinary field theory, the local minimum is unstable, namely the local minimum decays to the global minimum. How long is the lifetime of the unstable vacuum? In the end of this section, we discuss the lifetime of the local minimum by the instanton calculation, [6, 7, 8] which gives

$$\tau \sim \frac{1}{V\Lambda^4} e^{S(\bar{\phi})}, \quad (3.1)$$

where V and Λ are the volume of the universe and the typical scale of the potential of the scalar ϕ . Here $\bar{\phi}$ is an instanton solution and $S(\phi)$ is the action of the scalar field ϕ . Since the lifetime of the local minimum should be longer than the age of our universe $\tau_0 \sim 10^{20}$ seconds, we get the condition

$$10^{200} \frac{\Lambda^4}{(GeV)^4} \leq e^{S(\bar{\phi})}. \quad (3.2)$$

When the difference of the potential energy between the two minimum is much smaller than the typical scale Λ^4 , then we get

$$S(\bar{\phi}) \sim 2\pi^2 O(1)/\epsilon^3, \quad (3.3)$$

where $\Delta V \sim \epsilon\Lambda^4$. Therefore $\epsilon \sim 1/10$ is small enough to satisfy the above condition. This situation seems not to require strong fine tuning and is not so difficult to be satisfied. It is not natural that a scalar potential has no local minimum with the above feature. Therefore we think that it is not unnatural that we are living in a false vacuum.

4 Primordial black hole

The black hole which is induced in the early universe is called primordial black hole. In our situation, some bubble universes may become the primordial black holes. But how heavy primordial black holes are induced and how many black holes are there in our universe? Generally it is not so easy to give a concrete conclusion on the formation of the black holes at the phase transition. In the literature, the formation of the black holes has been discussed in the various cases with various assumptions. [9, 10, 11, 12] However, for the situation that the size of coexisting bubbles is around the horizon length $d_H \sim O(1/H)$, which can be reached by the light from the beginning of the universe, we can make a simple argument. It is consistent with the previous assumption that Friedmann equation can be applied to the evolution of the bubble universes.

The mass of the black holes can be estimated from the size of the bubbles and the energy density. The energy density is given by

$$\rho \sim T_{RC}^4, \quad (4.1)$$

where T_{RC} is the temperature at which the bubble begin to recollapse. Since in the most cases we discussed T_{RC} is the same order of the temperature of the phase transition T_{PT} , we use T_{PT} in the following argument. We can estimate the order of the mass of the primordial black hole by using the temperature and Planck scale M_P as

$$M_{BH} \sim d_H^3 \rho \sim M_P^3 / T_{PT}^2. \quad (4.2)$$

Here we use the relation $H^2 \sim G\rho$. The Schwarzschild radius $R_S = 2GM_{BH}$ for the black hole becomes

$$R_S \sim \frac{1}{H}. \quad (4.3)$$

It is well-known but surprising that the Schwarzschild radius is the same order of the horizon length which is the size of the bubbles. Therefore it is expected that the shrinking bubbles become black holes soon after the density outside the bubble becomes smaller than the inside one. It is natural that the recollapsing bubbles become black holes.

On the other hand, black holes are evaporating by Hawking radiation. [13] The lifetime is given by

$$\tau_{BH} \sim \frac{2560\pi}{g_*} G^2 M_{BH}^3 \sim \frac{2560\pi}{g_*} \frac{M_P^5}{T_{PT}^6}. \quad (4.4)$$

Here g_* is the degrees of freedom of the particles at the Hawking temperature $T_H = 1/(8\pi GM_{BH})$. From the table 2

| T_{PT} (GeV) | 1 | 100 | 10^9 | 10^{12} | 10^{19} |
|-----------------|--------------------------------|---------------|--------------------------|-----------|---------------|
| M_{BH} (g) | $O(10^{33}) \sim O(M_{Solar})$ | $O(10^{29})$ | $O(10^{15})$ | $O(10^9)$ | $O(10^{-5})$ |
| T_H (GeV) | $O(10^{-21})$ | $O(10^{-17})$ | $O(10^{-3})$ | $O(10^3)$ | $O(10^{17})$ |
| τ_{BH} (s) | $O(10^{74})$ | $O(10^{62})$ | $O(10^{20}) \sim \tau_0$ | $O(1)$ | $O(10^{-42})$ |

table 2. Here M_{Solar} is the solar mass and τ_0 is the age of our universe.

we can easily find that the primordial black holes may be seen in our universe if the temperature of the phase transition is less than 10^9 GeV. This is because the age of our universe is around $10^{20}s$. The black holes caused by the phase transition above 10^9 GeV evaporate until now, though there may exist some remnant of the black holes. Notice that the mass of the black hole from the QCD

phase transition is around the solar mass M_{Solar} which is the same order as the mass of the massive compact halo objects (MACHO) found by using gravity lensing effect. [14]

The number density is difficult to be estimated, because it is strongly dependent on the expanding rate of our universe, the shape of the potential, the features of the phase transition and the initial conditions. But here we roughly estimate the number density, though this estimation is not so reliable in general cases. The number density at the phase transition can be naively estimated as

$$n_{BH}(t_{PT}) \sim \frac{1}{d_H^3} \sim H^3 \sim \frac{T_{PT}^6}{M_P^3}. \quad (4.5)$$

Since the black holes are regarded as the non-relativistic objects, the number density after the phase transition is

$$n_{BH}(t) = n_{BH}(t_{PT}) \left(\frac{R(t_{PT})}{R(t)} \right)^3. \quad (4.6)$$

In the case 2, the density of the black holes dominates the energy density soon, which means that the universe becomes matter dominant. Since usually we think that the temperature of the phase transition is much larger than $T \sim 5.5$ eV at which matter density becomes almost equal to the radiation density, in order to regard the universe as our universe, we need the evaporation of the black holes (i.e., the scale of the phase transition is larger than 10^9 GeV) or another inflation realizing smaller density of the black holes.

In the case 1, after the inflation ends, the number density is estimated as

$$n_{BH}(t) = n_{BH}(t_{PT}) e^{-3Ht_E} \left(\frac{R(t_E)}{R(t)} \right)^3, \quad (4.7)$$

where t_E is the time when the inflation ends. Unfortunately in this case it is almost impossible to see the black holes in our universe. This is because there is only a few black hole at the most in our universe within the present horizon since the present universe is almost flat.

The interesting case is the intermediate case between cases 1 and 2. Namely the local minimum has a positive cosmological constant and the global minimum has a negative cosmological constant. Suppose that before the phase transition, $\Omega = 1$ has already been realized by another inflation etc. In this case, the bubbles with the global minimum shrink and become black holes soon, and the bubbles with the local minimum is inflating. The number density can be estimated as in the eq. (4.7), and we can realize any number density by tuning the time the inflation stops. Therefore we may see the primordial black holes caused by the phase transition and recollapsing process in our universe. In this scenario, the primordial black holes can be dark matter candidates for any level. Though it is interesting to examine the possibility that the small primordial black hole becomes the dark matter, it is beyond the subject in this paper.

5 Cosmological SUSY breaking

In this section, we examine applications of the above results to the particle physics. If it is usual that the local minimum is selected dynamically by disappearance of the global minimum, it must be interesting to examine the possibility that we are living in the local minimum more seriously. Of course, such a possibility has been already examined, because the local minimum can have longer lifetime than the age of our universe. But for the most of people, such a passive reason seems not to be enough to take the possibility seriously. Actually, many people have discussed the conditions for realizing the expected vacuum as the global minimum [15] and only the structure of the space of the global minimum (moduli space) in the supersymmetric field theories. However, as we argued, the universe with the global minimum may collapse and disappear in the history of the universe. Though the results is dependent on the initial conditions, it must be important to examine the possibility that we are living in the local minimum because we may have to stay on the local minimum. The examination of the possibility may be crucial for finding the real vacuum. For example, in the string theory, to realize the dilaton stability is one of the interesting problem to be solved. By changing the Kähler potential of the dilaton, we can stabilize the dilaton vacuum though this vacuum is often at the local minimum. If the local minimum is cosmologically selected, we do not have to mind why we are not living in the global minimum, which is too weak to survive. Moreover we think it important to examine the possibility in various phase transition, for examples, in the QCD phase transition, the electroweak phase transition and GUT phase transition. But here we discuss only the SUSY breaking.

If the nature has SUSY, scalars must exist. It seems to be natural that the scalar potential has several local minima which have features for longer life than the age of our universe. Since cosmology prefers local minimum, it is natural that such local minima is selected in the history of the universe. Since only the global minimum keeps the SUSY in the context of the global SUSY theories, in the theory with local minimum SUSY is spontaneously broken. Notice that SUSY is spontaneously broken even if SUSY vacua exist. Therefore SUSY is broken by cosmology.

Where is the natural scale for the SUSY breaking? If a theory has SUSY at a scale, then the theory has generally a scalar potential. It is natural that the potential has the local minima which break SUSY. The natural SUSY breaking scale is the scale. If the theory has a SUSY at the Planck scale, then it is natural that SUSY is broken at the Planck scale. Of course, such an argument does not reject the low energy SUSY. The theory may have no local minima with long life. Even if it has local minima, the initial conditions may allow the universe with vanishing cosmological constant to expand.

By using cosmological SUSY breaking, we can make simpler models in which SUSY is spontaneously broken. The point is that these models are allowed to

have SUSY vacua. Even if the models have SUSY vacua, cosmology prefers SUSY breaking local minima. We examine a simple model with gauge mediated SUSY breaking.

Several years ago, Izawa-Yanagida [16] and Intriligator-Thomas [17] proposed one of the simplest dynamical SUSY breaking model which has a SUSY SU(2) gauge group with four doublet chiral superfields $Q_i (i = 1, \dots, 4)$ and several singlets S and $S^a (a = 1, \dots, 5)$. Here the suffix i is a flavor index and a is the index of five dimensional representation of SP(4) global symmetry which they adopted as the global symmetry of the superpotential

$$W = ySQQ + \bar{y}S^a(QQ)_a, \quad (5.1)$$

where (QQ) and $(QQ)_a$ are one and five dimensional representations of SP(4) which are formed from a suitable combination of $Q_i Q_j$. The effective superpotential is given by

$$W_{eff} = y\Lambda SV + \bar{y}\Lambda S^a V_a \quad (5.2)$$

with constraint $V^2 + V_a^2 - \Lambda^2 = 0$. Here the composite meson fields $V \sim (QQ)/\Lambda$ and $V_a \sim (QQ)_a/\Lambda$ are the low energy degrees of freedom and Λ is a dynamical scale of the SU(2) gauge theory. When $y < \bar{y}$, the condensation

$$\langle V \rangle = \Lambda, \langle V_a \rangle = 0 \quad (5.3)$$

is realized at the global minimum, and the effective potential is approximately rewritten as

$$W = y\Lambda^2 S. \quad (5.4)$$

The F-term of the S field F_S is $y\Lambda^2 \neq 0$, which means that the SUSY is spontaneously broken. Notice that if the Kähler potential of the S field is minimal one, then the vacuum expectation value (VEV) of the S field is undetermined, namely, the potential has a flat direction.

In order to mediate the SUSY breaking effect to the ordinary gauginos, squarks and sleptons via the standard model gauge interaction (gauge mediation), [18] we introduce vector-like messenger fields q and \bar{q} which has quantum numbers of the standard gauge group and the superpotential

$$W = \lambda S \bar{q} q. \quad (5.5)$$

If the vacuum expectation values of these fields remain unchanged by this deformation, namely,

$$\langle V \rangle = \Lambda, \langle V_a \rangle = 0, \langle q \rangle = \langle \bar{q} \rangle = 0, \quad (5.6)$$

the supersymmetry is broken again. If we take $\langle S \rangle^2 > F_S$, then ordinary gauge mediation calculation gives the gaugino masses

$$M_a \sim \frac{\alpha_a}{4\pi} \frac{\langle F_S \rangle}{\langle S \rangle} \quad (5.7)$$

via one loop diagram and scalar fermion masses

$$\tilde{m}_f^2 \sim \sum_a \left(\frac{\alpha_a}{4\pi} \right)^2 \left(\frac{\langle F_S \rangle}{\langle S \rangle} \right)^2 \quad (5.8)$$

via two loop diagram. Here $\alpha_a \equiv g_a^2/(4\pi)$ and g_a are the gauge couplings of the standard model. Notice that the order of the SUSY breaking mass scales of sfermions is the same as of gauginos. This mediation mechanism can give the realistic mass pattern to the superpartners.

This naive model is very simple but unfortunately by adding the above superpotential, SUSY vacua appear, namely the F-term equation

$$y\Lambda^2 - \lambda\bar{q}q = 0 \quad (5.9)$$

can be satisfied by taking non vanishing vacuum expectation value of $\bar{q}q$. Therefore usually this model has not been examined as a dynamical SUSY breaking model. Several attempts to make the SUSY vacua disappear have been made in the literature. [19, 20] However, since cosmology prefers local minima, we do not mind that the SUSY vacua exist. What is important here is that the expected vacuum should be at least local minimum and the lifetime of the vacuum is longer than the age of our universe.

The potential under the superpotential and the minimum Kähler potential is

$$V = |y\Lambda^2 - \lambda\bar{q}q|^2 + |\lambda S|^2(|q|^2 + |\bar{q}|^2) + \text{Dterm} \quad (5.10)$$

which has a flat direction along the direction $q = \bar{q} = 0$. Along the flat direction, when $|\lambda S|^2 > y\Lambda^2$, the mass square matrix of the fields q and \bar{q} has only positive eigenvalues, on the other hand, when $|\lambda S|^2 < y\Lambda^2$, it has negative eigenvalues. Therefore the expected vacuum (5.6) is not even a local minimum. However, generally, the Kähler potential can be deformed by quantum corrections from the minimal one. Then the expected vacuum may be a local minimum. Actually if the Kähler potential of the S field is corrected as

$$K = |S|^2 + \eta \frac{|yS|^4}{\Lambda^2} (\eta > 0) \quad (5.11)$$

by the strong gauge interaction, then the effective potential along the flat direction $q = \bar{q} = 0$ is given by

$$V_{eff} \sim |y|^2 \Lambda^4 (1 - 4 \frac{\eta}{\Lambda^2} |y|^4 |S|^2). \quad (5.12)$$

Since the effective potential is lifted in the large $|S|$ region because the Yukawa coupling y grows with the scale S , [17, 21] the potential along the flat direction has a minimum. If the condition $|\lambda S|^2 > y\Lambda^2$ is satisfied at the minimum, the minimum is a local minimum in the whole potential. Since cosmology prefers local

minima, the local vacuum can be selected cosmologically if the lifetime is longer than the age of our universe, which is strongly dependent on the parameters and the vacuum expectation value.

This is a simple model in which cosmological SUSY breaking happens, though we may need to introduce another singlet or non-renormalizable terms in order to get the realistic scale of the supersymmetric Higgs mass term.

In this cosmological SUSY breaking scenario, bubbles with SUSY vacua shrink and become black holes. It is interesting that the mass of the black holes is determined by the SUSY breaking scale (see table 2). Therefore by observing the small black holes in our universe, we may measure the SUSY breaking scale. Since the SUSY breaking scale for the gauge mediation scenario is usually between 10^5 GeV and 10^{10} GeV, the mass scale of the black holes is $10^{13}\text{g} < M_{BH} < 10^{23}\text{g}$. Though the black holes with lighter mass than 10^{15} g evaporate till now, it is interesting that the most of the gauge mediated SUSY breaking models ($10^5\text{GeV} < \sqrt{F_S} < 10^9\text{GeV}$) produce the small black holes with longer lifetime than the age of our universe and we can know the SUSY breaking scale from the mass of the black holes.

6 Discussion

Coleman and DeLuccia have argued on the probability of the first order phase transition including the gravity by finding the O(4) symmetric instanton solution of the scalar field theory with gravity. Since the solution has O(4) symmetry, the bubble wall velocity becomes light velocity soon and the bubbles keep expanding. Even when the global minimum has negative cosmological constant, they found the O(4) symmetric solution, therefore the bubble keeps expanding. They concluded that the observer outside the bubble sees the bubble expanding, on the other hand, the space inside the bubble is collapsing. Though this conclusion is strongly dependent on their assumption that the O(4) solution gives the smallest action, this seems to give a counter example of our assumption that the evolution of the bubble is determined by the Friedmann equation. Similar phenomenon is examined in a different situation at the first order phase transition. [9] Of course including finite temperature effects, the wall velocity may become much less than the light velocity (i.e., the solution has not O(4) symmetry). [22] However, it suggests that our calculation may be too naive. More reliable calculation including numerical estimation is needed, which is a future subject.

The determination of the vacuum is equivalent to the determination of the theory in a sense, in some cases the determination of the couplings. If we can construct a scenario in which only very limited vacua survive in the evolution of the universe, then we may be able to solve the fine tuning problems of couplings, scales et al., for examples, strong CP phase or Higgs mass, by using cosmology. This direction may be interesting, but this is also one of the future subjects.

The discussion in this paper is similar to the anthropic principle, which insists that physical parameters must be as they are, otherwise there does not exist creatures like human beings who think why physical parameters are like these. There are some difference between the usual human principle and our arguments. The biggest difference is that in our scenario other universes may be observed in our universe as black holes. In this sense, we think that our argument is similar to the theory of evolution for creatures, because we can observe a lot of fossils (black holes) as the remnants of the extinct creatures (recollapsing universes).

7 Summary

Under several assumptions we examined the several examples in which the vacuum is selected by recollapsing. It is interesting that the recollapsing universes become black holes which may be observed in our universe. The mass of the black holes can be roughly estimated in the situation that the bubble radius is around the horizon length, it is interesting that QCD phase transition may produce the black holes with solar mass, because MACHOs with solar mass have already been observed by gravity lensing effect.

It is also interesting that the local minimum is preferable to global minimum. If this result is applied to the SUSY theory, SUSY may be cosmologically broken even if the theory has SUSY global vacua. We made a simple model in which SUSY is cosmologically broken.

The assumptions we imposed in this paper may be too naive. But we believe that even if the assumptions are not satisfied in many cases, the vacuum selection by recollapsing can generally happen and tells us important clues to know why our world is as it is.

In the sense of the theory of the evolution for creatures, it may be considerable that bubbles with different physical theories fight each other and bubbles with theories which will make other bubbles disappear may survive. Such a consideration like the struggle for existence may give clearer features of the theories which survive the history of the universe.

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References

- [1] For a review, E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, 1990).
- [2] A. Linde, *Phys. Lett.* **B129** (1983) 177, *Phys. Lett.* **B162** (1985) 281, for a review, A. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, 1990).
- [3] A. Linde, *Phys. Lett.* **B327** (1994) 208;
A. Vilenkin, *Phys. Rev. Lett.* **72** (1994) 3137;
A. Linde and D. Linde, *Phys. Rev.* **D50** (1994) 2456.
- [4] K.-I. Izawa and T. Yanagida, hep-ph/9904426.
- [5] T. Banks, M. Berkooz and P.J. Steinhardt, *Phys. Rev.* **D52** (1995) 705,
T. Banks, hep-th/9601151.
- [6] A. Linde, *Phys. Lett.* **B100** (1981) 37.
- [7] S. Coleman, *Phys. Rev.* **D15** (1977) 2922.
- [8] S. Coleman and F. DeLuccia, *Phys. Rev.* **D21** (1980) 3305.
- [9] K. Sato, M. Sasaki, H. Kodama and K. Maeda, *Prog. Theor. Phys.* **65** (1981) 1443;
H. Kodama, M. Sasaki, K. Sato and K. Maeda, *Prog. Theor. Phys.* **66** (1981) 2052;
H. Kodama, M. Sasaki and K. Sato, *Prog. Theor. Phys.* **68** (1982) 1979.
- [10] S.W. Hawking, I.G. Moss and J.M. Stewart, *Phys. Rev.* **D26** (1982) 2681.
- [11] L.J. Hall and S.D.H. Hsu, *Phys. Rev. Lett.* **64** (1990) 2848.
- [12] K. Jedamzik, *Phys. Rev.* **D55** (1997) 5871;
K. Jedamzik and J.C. Niemeyer, *Phys. Rev.* **D59** (1999) 124014.
- [13] S. Hawking, *Nature* **248** (1974) 30; *Commun. Math. Phys.* **43** (1975) 199.
- [14] C. Alcock et al., astro-ph/0001435; *Astrophys. J.* **461** (1996) 84.
- [15] J.A. Casas, A. Lleyda and C. Muñoz, *Nucl. Phys.* **B471** (1996) 3.
H. Baer, M. Brihlik and D. Castaño, *Phys. Rev.* **D54** (1996) 6944.
- [16] K.-I. Izawa and T. Yanagida, *Prog. Theor. Phys.* **95** (1996) 829.
- [17] K. Intriligator and S. Thomas, *Nucl. Phys.* **B473** (1996) 121.

- [18] M. Dine and A.E. Nelson, *Phys. Rev.* **D48** (1993) 1277;
M. Dine, A.E. Nelson and Y. Shirman, *Phys. Rev.* **D51** (1995) 1362;
for review, see G.F. Giudice and R. Rattazzi, *Phys. Rept.* **322** (1999) 419.
- [19] T. Hotta, K.-I. Izawa and T. Yanagida, *Phys.Rev.* **D55** (1997) 415,
K.-I. Izawa, Y. Nomura, K. Tobe and T. Yanagida, *Phys.Rev.* **D56** (1997) 2886.
- [20] H. Murayama, *Phys. Rev. Lett.* **79** (1997) 18.
- [21] Y. Shirman, *Phys. Lett.* **389** (1996) 287.
- [22] A. Linde, *Nucl. Phys.* **B216** (1983) 421.